AS - 4064

B. Tech (3rd Sem) Exam, 2013 (Ind & Prod. Engg)

Theory of M/C

Section - A

I (i) c (ii) a (iii) c (iv) b (v) d

II (vi) Parallel

(vii) Straight line

(viii) Decreases

(ix) Pitch circle diameter

(x) Pickering

III

(xi) The perfect steering is achieved when all the four wheels are rolling perfectly under all conditions of running while taking turns, the condition of perfect rolling is satisfied if the axes of the front wheels when produced meet the rear wheel axis at one point, this point is the instantaneous centre of the vehicle.

Davis steering gear has sliding pairs which means more friction & easy wearing. The gear fulfills the fundamental eq^n of gearing in all the positions. However, due to easy wearing it becomes inaccurate after some time.

(xii) Kennedy's Theorem : If three planes bodies have relative motion among themselves, their I-centres must lie on a straight line.

Configuration Diagram: A M/C or a mechanism, represented by a skeleton or a line diagram.

(xiii) When power being transmitted exceeds the torque capacity, corresponding to limiting coefficient friction, the belt slips over the pulley.

The effect of slip is to decrease the speed of the belt on the driving shaft & to ~~decrease the~~ speed of the driven shaft.

Creep: As more length of belt approaches the driving pulley than the length that leaves, the belt slips back over the driving pulley. This slip is known as creep of the belt.

Positive Drives: Gear, ~~Belt drive~~

Non-Positive Drives: Chain drives

(xiv) Positive drive exist in a direct contact mechanism if the motion of the driving member compels the driven member to move. It can exist, when the common normal through the pt of contact must not pass through either or both of the centres of rotation.

Gears are +ve drives becoz teeth are placed on the contacting members & the resulting m/c members are called gears.

(xv) Porter & Proell both governors are modifications of simple centrifugal governors & both are dead wt. type governors. In case of Proell governor, the governor balls are attached on the extension of lower links instead of at the junction of lower & upper arms. The additional ~~dead~~ wt. on sleeve as compared to simple governor increases the speed of rotation at a given radii of rotation. The advantage of Proell over Porter type is that it needs smaller size governor balls for the same equilibrium speeds & radii of rotn, alternatively, it has lower equilibrium speed at given radius compared to Porter governor when the mass of the balls & the dead wt. are same. (3)

Section B

Unit - I

Q 2 Inversions of [redacted] DOUBLE SLIDER-CRANK CHAIN

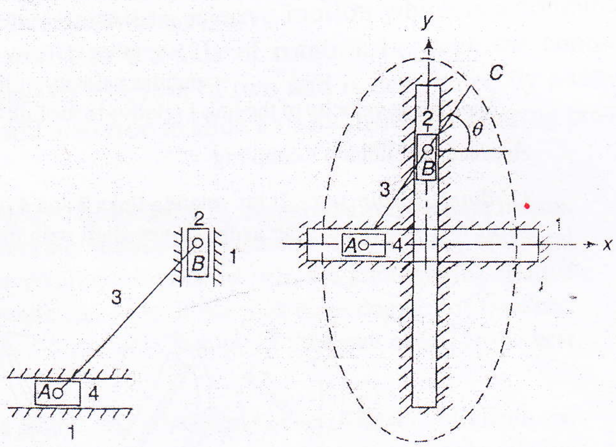
A four-bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as a double-slider-crank chain [redacted]. The following are its inversions.

First Inversion

This inversion is obtained when the link 1 is fixed and the two adjacent pairs 23 and 34 are turning pairs and the other two pairs 12 and 41 sliding pairs.

Application Elliptical trammel

Elliptical Trammel Figure [redacted](b) shows an elliptical trammel in which the fixed link 1 is in the form of guides for sliders 2 and 4. With the movement of the sliders, any point C on the link 3, except the midpoint of AB will trace an ellipse on a fixed plate. The midpoint of AB will trace a circle.



(a) (b)

[redacted]

Let at any instant, the link 3 make angle θ with the X-axis. Considering the displacements of the sliders from the centre of the trammel,

$$x = BC\cos\theta \text{ and } y = AC\sin\theta$$

$$\therefore \quad \frac{x}{BC} = \cos\theta \text{ and } \frac{y}{AC} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(BC)^2} + \frac{y^2}{(AC)^2} = \cos^2\theta + \sin^2\theta = 1$$

This is the equation of an ellipse. Therefore, the path traced by C is an ellipse with the semi-major and semi-minor axes being equal to AC and BC respectively.

When C is the midpoint of AB; $AC = BC$,

and

$$\frac{x^2}{(BC)^2} + \frac{y^2}{(AC)^2} = 1 \quad \text{or} \quad x^2 + y^2 = (AC)^2$$

which is the equation of a circle with AC (=BC) as the radius of the circle.

Second Inversion

If any of the slide-blocks of the first inversion is fixed, the second inversion of the double-slider-crank chain is obtained. When the link 4 is fixed, the end B of the crank 3 rotates about A and the link 1 reciprocates in the horizontal direction.

Application Scotch yoke

Scotch Yoke A scotch-yoke mechanism (Fig. 1.59) is used to convert the rotary motion into a sliding motion. As the crank 3 rotates, the horizontal portion of the link 1 slides or reciprocates in the fixed link 4.

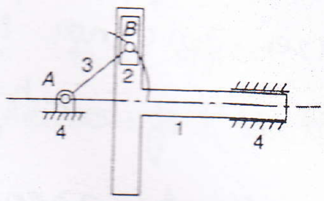


Fig. 1.59

Third Inversion

This inversion is obtained when the link 3 of the first inversion is fixed and the link 1 is free to move.

The rotation of the link 1 has been shown in Fig. 1.60 in which the full lines show the initial position. With rotation of the link 4 through 45° in the clockwise direction, the links 1 and 2 rotate through the same angle whereas the midpoint of the link 1 rotates through 90° in a circle with the length of link 3 as diameter. Thus, the angular velocity of the midpoint of the link 1 is twice that of links 2 and 4.

The sliding velocity of the link 1 relative to the link 4 will be maximum when the midpoint of the link 1 is at the axis of the link 4. In this position, the sliding velocity is equal to the tangential velocity of the midpoint of the link 1.

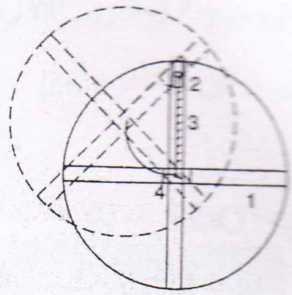


Fig. 1.60

Maximum sliding velocity = tangential velocity of midpoint of the link 1
= angular velocity of midpoint of the link 1 × radius
= (2 × angular velocity of the link 4) × (distance between axes of links 2 and 4)/2
= angular velocity of link 4 × distance between axes of links 2 and 4

The sliding velocity of the link 1 relative to the link 4 is zero when the midpoint of 1 is on the axis of the link 2.

Application Oldham's coupling

Oldham's Coupling If the rotating links 2 and 4 of the mechanism are replaced by two shafts, one can act as the driver and the other as the driven shaft with their axes at the pivots of links 2 and 4.

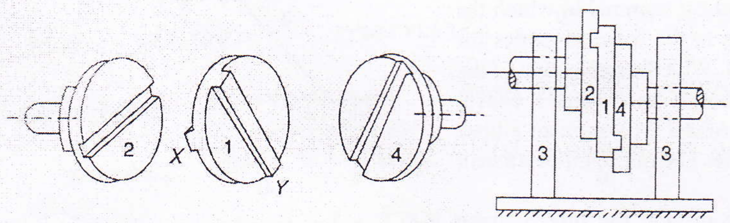


Fig. 1.61

Figure 1.61 shows an actual Oldham's coupling which is used to connect two parallel shafts when the distance between their axes is small. The two shafts have flanges at the ends and are supported in the fixed bearings representing the link 3. In the flange 2, a slot is cut in which the tongue X of the link 1 is fitted and has a sliding motion. Link 1 is made circular and has another tongue Y at right angles to the first and which fits in the recess of the flange of the shaft 4. Thus, the intermediate link 1 slides in the two slots in the two flanges while having the rotary motion.

As mentioned earlier, the midpoint of the intermediate piece describes a circle with distance between the axes of the shafts as diameter. The maximum sliding velocity of each tongue in the slot will be the peripheral velocity of the midpoint of the intermediate disc along the circular path.

Maximum sliding velocity = peripheral velocity along the circular path
= angular velocity of shaft × distance between shafts

Q2 (a) Classification of Pairs Based on Type of Relative Motion

The relative motion of a point on one element relative to the other on mating element can be that of turning, sliding, screw (helical direction), planar, cylindrical or spherical. The controlling factor that determines the relative motions allowed by a given joint is the shapes of the mating surfaces or elements. Each type of joint has its own characteristic shapes for the elements, and each permits a particular type of motion, which is determined by the possible ways in which these elemental surfaces can move with respect to each other. The shapes of mating elemental surfaces restrict the totally arbitrary motion of two unconnected links to some prescribed type of relative motion.

(i) *Turning Pair*. (Also called a hinge, a pin joint or a revolute pair). This is the most common type of kinematic pair and is designated by the letter R.
A pin joint has cylindrical element surfaces and assuming that the links cannot slide axially, these surfaces permit relative motion of rotation only. A pin joint allows the two connected links to experience relative rotation about the pin centre. Thus, the pair permits only one degree of freedom. Kinematic pairs, marked R in Fig. 2.2, represent turning or revolute pairs. Thus, the pair at piston pin, the pair at crank pin and the pair formed by rotating crank-shaft in bearing are all examples of turning pairs.

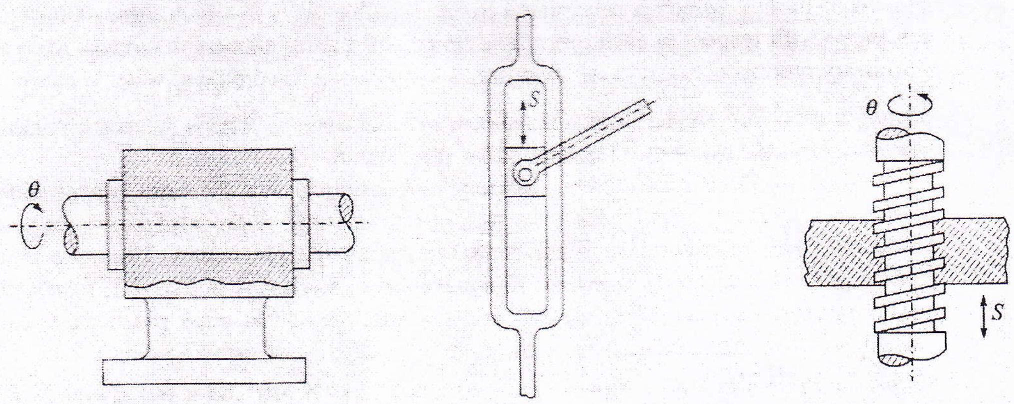
(ii) *Sliding or Prismatic Pair.* This is also a common type of pair and is designated as P. This type of pair permits relative motion of sliding only in one direction (along a line) and as such has only one degree of freedom. Pairs between piston and cylinder, cross-head and guides, die-block and slot of slotted lever are all examples of sliding pairs.

(iii) *Screw Pair.* This pair permits a relative motion between concident points, on mating elements, along a helix curve. Both axial sliding and rotational motions are involved. But as the sliding and rotational motions are related through helix angle α, the pair has only one degree of freedom. The pair is commonly designated by the letter S. Examples of such pairs are to be found in translatory screws operating against rotating nuts to transmit large forces at comparatively low speed, *e.g.* in screw-jacks, screw-presses, valves and pressing screw of rolling mills. Other examples are rotating lead screws operating in nuts to transmit motion accurately as in lathes, machine tools, measuring instruments, etc.

(iv) *Cylindrical Pair.* A cylindrical pair permits a relative motion which is a combination of rotation θ and translation s parallel to the axis of rotation between the contacting elements. The pair has thus a degree of freedom of two and is designated by a letter C. A shaft free to rotate in a bearing and also free to slide axially inside the bearing provides example of a cylindrical pair.

(v) *Globular or Spherical Pair.* Designated by the letter G, the pair permits relative motion such that coincident points on working surfaces of elements move along spherical surface. In other words, for a given position of spherical pair, the joint permits relative rotation about three mutually perpendicular axes. It has thus three degrees of freedom. A ball and socket joint (*e.g.*, the shoulder joint at arm-pit of a human being) is the best example of spherical pair.

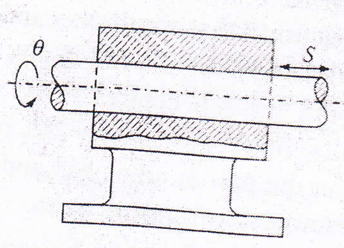
(vi) *Flat pair (Planar Pair).* A flat or planar pair is seldom, if ever, found in mechanisms. The pair permits a planar relative motion between contacting elements. This relative motion can be described in terms of two translatory motions in x and y directions and a rotation θ about third direction z, – x, y, z being mutually perpendicular directions. The pair is designated as F and has a degree of freedom of 3.
All the above six types of pairs, illustrated in Fig. 2.6, are representative of a particular class.



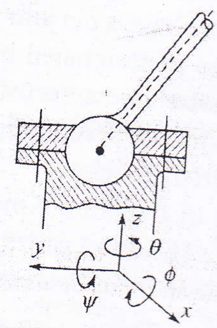
(a) Turning pair (revolute) R
D.O.F. = 1

(b) Prismatic (slider) pair P
D.O.F. = 1

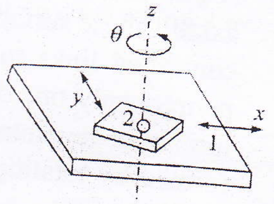
(c) Screw/helical pair S
D.O.F. = 1



(d) Cylindrical pair C
D.O.F. = 2



(e) Globular (spherical) pair G
D.O.F. = 3



(f) Flat (planar) pair F
D.O.F. = 3

(vii) *Rolling Pair.* When surfaces of mating elements have a relative motion of rolling, the pair is called a rolling pair. Castor wheel of trolleys, ball and roller bearings, wheels of locomotive/wagon and rail are a few examples of this type.

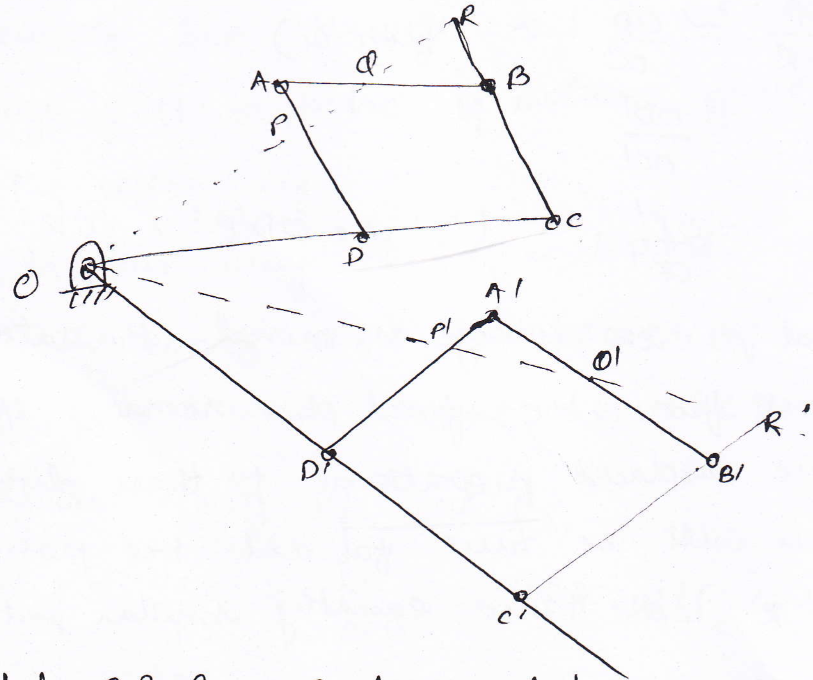
Q 2.(b) Pantograph -

(½) ~~Derivation~~ Is a four bar linkage used to produce paths exactly similar to the ones traced out by a pt. on the linkage. The paths so produced r, usually on an enlarged or reduced scale & may be straight or curved ones.

The four links of a pantograph r arranged in such a way that a ||gm ABCD is formed.

Thus AB = DC & BC = AD

If some pt. O in one of the links is made fixed & three pts. P, Q & R on the other three links & located in such a way that OPQR is a straight line, it can be shown that the points P, Q & R always move ||el & similar to each other over any path, straight or curved. Their motions will be proportional to their distances from the fixed pt.



Let O, P, Q & R lie on links CD, DA, AB & BC.

ABCD is the initial assumed position.

Let the linkage be moved to another position so that A moves to A', B to B' & so on.

In Δ ODP & OCR,

O, P & R lie on a straight line & thus OP & OR coincide.

$\angle DOP = \angle COR$ (common $\angle$s)

$\angle ODP = \angle OCR$ ($\because DP \parallel CR$)

the Δs are similar &

$$\frac{OD}{OC} = \frac{OP}{OR} = \frac{DP}{CR} \qquad \text{— (i)}$$

$A'B' = AB = DC = D'C'$

$B'C' = BC = AD = A'D'$

$\therefore$ A'B'C'D' is again a ||gm

In Δs $OD'P'$ & $OC'R'$

$$\frac{OD'}{OC'} = \frac{OP}{OC} = \frac{DP}{CR} = \frac{D'P'}{C'R'} \qquad \text{(from (i))}$$

& $\angle OD'P' = \angle OC'R'$ ($D'P' \parallel C'B'$ as $A'B'C'D'$ is a llgm)

Thus ; the Δs r similar

$\angle D'OP' = \angle C'OR'$

or O, P' & R' lie on a straight line.

Now $\dfrac{OP}{OR} = \dfrac{OD}{OC}$ from (i)

$= \dfrac{OD'}{OC'}$

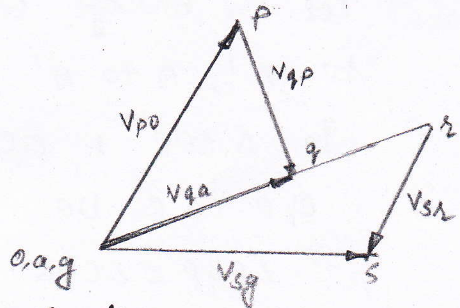
$= \dfrac{OP'}{OR'}$ ($\because$ Δs $OD'P'$ & $OC'R'$ r similar)

This shows that as the linkage is moved, the ratio of the distances of P & R from the fixed pt. remains the same, or the two pts. are displaced proportional to their distances from the fixed pt. This will be true for all the positions of the links. Thus P & R will trace exactly similar paths.

$\parallel$ly It can also be proved that P & Q trace similar paths. Thus P, Q & R trace similar paths when the linkage is given motion.

UNIT- II

Q3:



$$\omega_{PO} = \frac{2\pi \times 120}{60} = 22 \text{ rad/s}$$

Draw the configuratⁿ to a suitable scale.

The velocity vector eqⁿ for the mechanism OPA,

$V_{qa} = V_{po} + V_{qp}$ or $q.aq = op + pq$

In this eqⁿ,

V_{po} or $op = \omega \cdot OP = 22 \times 0.2 = 4.4$ m/s

Take the vector V_{po} which is fully known. Velocity diagram (c)

$V_{qp} \parallel AR$, draw a line $\parallel$ to AR through p;

$V_{qa} \perp AR$, → ———— ⊥ → ———— a (or o).

The intersectn locates the pt. q. Locate the pt. r on the vector aq produced s. $\frac{ar}{aq} = \frac{AR}{AQ}$.

Draw a line through r ⊥ to RS for the vector V_{sr} & a line through g. ∥ to the line of motion of the slider S on the guide G. for the vector V_{sg}.

In this way pt. s is located.

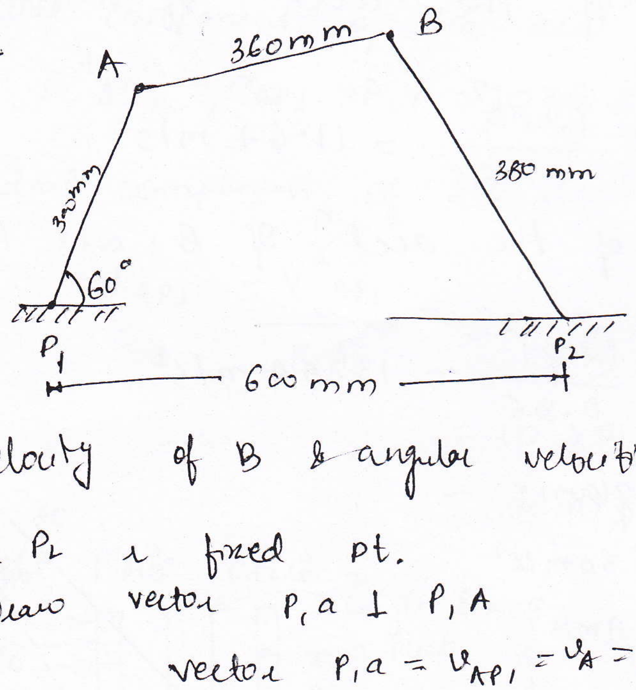
The velocity of the ram $S = os (or\, gs) = 4.5\, m/s$

It is towards right for the position of the crank.

Angular velocity of link RS,

$$\omega_{rs} = \frac{v_{rs}}{RS} = \frac{1.4}{0.3} = 4.67\ rad/s\ cw.$$

OR



Given $\omega_{AP_1} = 10\ rad/s$

$\alpha_{AP_1} = 30\ rad/s^2$

$P_1A = 300mm = 0.3m$

$R_2B = AB \doteq 360mm = 0.36m$

Velocity of B & angular velocities of P_2B & AB

1. $P_1 P_2$ r fixed pt.

   Draw vector $p_1a \perp P_1A$

   vector $p_1a = v_{AP_1} = v_A = 3\ m/s$

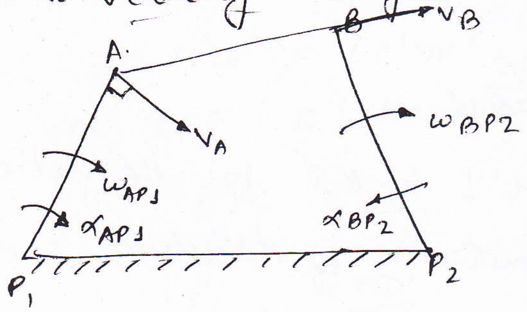
2. $v_{BP2} = v_B =$ vector $p_2b \doteq 2.2\ m/s$

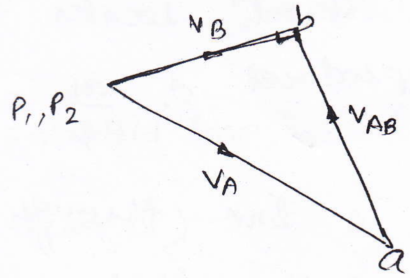
   $v_{BA} =$ vector $ab = 2.03\ m/s$

Angular velocity of P_2B, $\omega_{P_2B} = \frac{v_{BP_2}}{P_2B} = \frac{2.2}{0.36} = 6.1$ rad/s

" ——————— AB, $\omega_{AB} = \frac{v_{BA}}{AB} = \frac{2.05}{0.36} = 5.7$ rad/s (ccw)

~~Accl^n~~ Velocity Diagram





Accl^n of B & angular accl^n of P_2B & AB

Tangential component of the accl^n of A wrt P_1

$$a^t_{AP1} = \alpha_{AP1} \times P_1A = 30 \times 0.3 = 9 \text{ m/s}^2$$

Radial component of the accl^n of A wrt P_1

$$a^r_{AP1} = \frac{V^2_{AP1}}{P_1A} = \omega^2_{AP1} \times P_1A = 10^2 \times 0.3 = 30 \text{ m/s}^2$$

Radial component of the accl^n of B wrt A

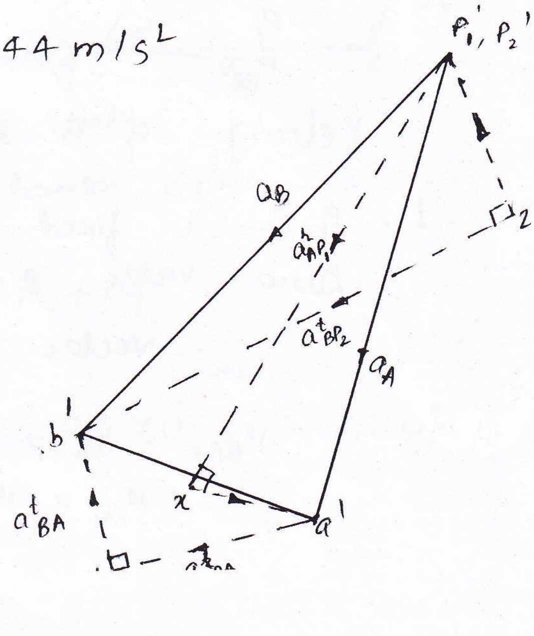
$$a^r_{BA} = \frac{V^2_{BA}}{AB} = \frac{(2.05)^2}{0.36} = 11.67 \text{ m/s}^2$$

Radial component of the accl^n of B wrt P_2

$$a^r_{BP_2} = \frac{V^2_{BP_2}}{P_2B} = \frac{(2.2)^2}{0.36} = 13.44 \text{ m/s}^2$$

The Accl^n Diagram:

1. vector $P_1'x = a^t_{AP1} = 30$ m/s^2
2. Vector $xa' = a^t_{AP1} = 9$ m/s^2
3. $a_A = a_{AP1} = 31.6$ m/s^2
4. vector $a'y = a^r_{BA} = 11.67$ m/s^2
5. vector $P_2'z = a^r_{BP_2} = 13.44$ m/s^2



vectors yb' & zb* intersect at b'. (21)

$a_{BP_2} = a_{B} =$ vector $P_2'b = 29.6\ m/s^2$

vector $yb' = a^t_{BA} = 13.6\ m/s^2$

〃 — $zb' = a^t_{BP_2} = 26.6\ m/s^2$

Angular accl^n of P_2B

$$\alpha_{P_2B} = \frac{a^t_{BP_2}}{P_2B} = \frac{26.6}{0.36} = 73.8\ rad/s^2\ (ccw)$$

angular accl^n of AB, $\alpha_{AB} = \frac{a^t_{BA}}{AB} = \frac{13.6}{0.36} = 38.8\ rad/s^2$ (ccw)

Angular velocity of P_2B

$$\omega_{2B} = \frac{v_{BP_2}}{P_2B} = \frac{2.2}{0.36} = 6.1\ rad/s\ (ccw)$$

Angular velocity of AB,

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{2.05}{0.36} = 5.7\ rad/s\ (ccw)$$

Accl^n of B & angular accl^n of P_2B & AB

tangential component of the accl^n of A wrt P_1

$$a^t_{AP_1} = \alpha_{AP_1} \times P_1A = 30 \times 0.3 = 9\ m/s^2$$

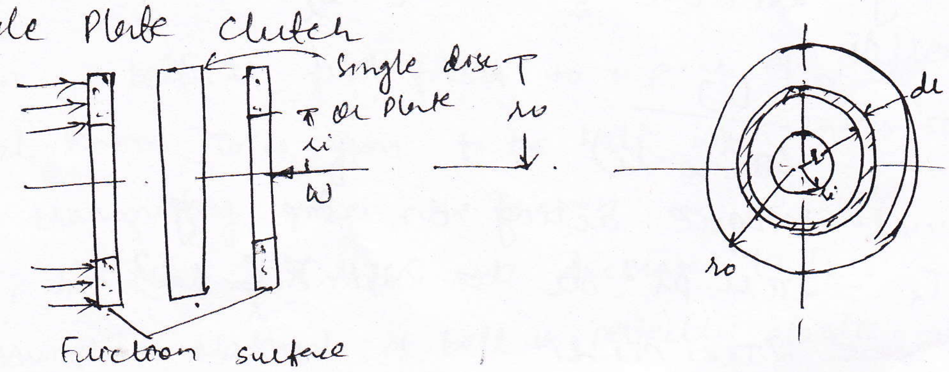
Radial component of the accl^n of A wrt P_1

$$a^r_{AP_1} = \frac{V^2_{AP_1}}{P_1A} = \omega^2_{AP_1} \times P_1A$$

$$= 10^2 \times 0.3$$

$$= 30\ m/s^2$$

Q 4 (a) Single Plate clutch



Considering two friction surfaces, maintained in cont by an axial thrust W

T = torque transmitted by the clutch

p = intensity of axial pressure with which the contact su are held together.

r_i & r_o = Internal & External radii of friction faces.

μ = Coeff. of friction

Consider an elementary ring of radius r and thickness dr

Area of contact surface or friction surface $= 2\pi r\,dr$

Normal or axial force on the ring, δW = press × Area

$= p \times 2\pi r\,dr$

Frictional force on the ring acting tangentially at radius r, $F_r = \mu \cdot \delta W = \mu p \times 2\pi r\,dr$

Frictional torque acting on the ring, $T_r = F_r \times r$

$= \mu p \times 2\pi r \cdot dr \times r = 2\pi \times \mu p r^2 dr$

Uniform Wear

Let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, $\therefore pr = C \Rightarrow p = \frac{C}{r}$

& the normal force on the ring, $\delta W = p \times 2\pi r\,dr$

$= \frac{C}{r} \times 2\pi r\,dr = 2\pi C\,dr$

$\therefore$ Total force acting on the friction surface,

$$W = \int_{r_i}^{r_o} 2\pi C\,dr = 2\pi C\,[r]_{r_i}^{r_o} = 2\pi C(r_o - r_i)$$

$$C = \frac{W}{2\pi(r_o - r_i)}$$

Frictional torque acting on the ring,

$T_r = 2\pi \mu \cdot p r^2\,dr = 2\pi\mu \times \frac{C}{r} \times r^2 dr$

$= 2\pi\mu\, C \cdot r\,dr$

Total frictional torque on the friction surface

$$T = \int_{ri}^{ro} 2\pi\mu \cdot c \cdot r\,dr$$

$$= 2\pi\mu \cdot c \left[\frac{ro^2 - ri^2}{2}\right] = \pi\mu \cdot c\,(ro^2 - r_i^2)$$

$$= \pi\mu \times \frac{W}{2\pi(ro - ri)}\,(ro^2 - ri^2)$$

$$T = \frac{\mu W}{2}(ro + ri) = \mu W R_m$$

where $R_m = \frac{ro + ri}{2}$ = Mean Radius

Q4 (b) ***Solution:*** Given: $n_o = 3$; $n_I = 2$; $r_1 = \frac{24}{2} = 12$ cm; $r_2 = \frac{12}{2} = 6$ cm

$\mu = 0.3$; $N = 1575$ r.p.m.; Power = 25 kw.

Number of pairs of active surfaces $= (n_o + n_I) - 1 = 3 + 2 - 1 = 4$.

The friction torque T_f is given by

$$25000 = \frac{2\pi N T_f}{60}$$

$$T_f = \frac{(25 \times 1000) \times 60}{2\pi \times 1575} = 151.6 \text{ N·m.}$$

For uniform wear rate,

$$T_f = \frac{1}{2}\mu W (n_a)(r_1 + r_2)$$

$$151.6 \times 10^2 = \frac{1}{2}(0.3)W(4)(12 + 6)$$

Therefore $W = \frac{151.6 \times 100 \times 2}{0.3 \times 4 \times 18} = 1403.7$ N

Again $W = 2\pi c(r_1 - r_2)$

where $c = p_2 r_2 = (6p_2)$

Hence $1403.7 = 2\pi(6p_2)(12 - 6)$

Hence $p_2 = \frac{1403.7}{2\pi \times 6 \times 6}$ 6.206 N/cm^2

The maximum pressure intensity $p_2 = 6.206$ N/cm^2 $= 6.206 \times 10^4$ N/m^2 $= 62.06$ KPa

Q4 (c) Initial tension of the belt, $T_o = \frac{T_1 + T_2}{2}$

When a belt is first fitted to a pair of pulleys, an initial tension To is given to the belt when the system is stationary. When transmitting power, the tension on the tight side increases to T_1 & that on slack side decreases to T_2.

Assumpt^n: Material of belt is perfectly elastic.

Solⁿ Speed of driving pulley, $N_1 = 750$ rpm (electric m

" —— driven " , $N_2 = 300$ " —

larger pulley driven pulley & $D = 800$ mm

$$\frac{d}{D} = \frac{N_2}{N_1}$$

$$\frac{d}{800} = \frac{300}{750} \quad ; \quad d = 320\,mm$$

$$r = 160\,mm$$

Mass of belt 1 m length = area × length × density

$$= 350 \times 10^{-6} \times 1 \times 900 = 0.315\ kg$$

Centrifugal tension, $T_c = mv^2 = 0.315 \times 30^2 = 283.5\,N$

Maximum " —— in the belt, $T = \sigma \times area = 2.2 \times 350 = 770\,N$

$$T_1 = T - T_c = 770 - 283.6$$

$$T_1 = 486.5\ N$$

$$\theta = \pi - 2\sin^{-1}\left(\frac{R-r}{C}\right)$$

$$= \pi - 2\sin^{-1}\left(\frac{400-160}{1500}\right) = 2.82\ rad$$

$$\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha} = \cancel{0.2}\, e^{0.28 \times 2.82/\sin 19^\circ}$$

$$\frac{T_1}{T_2} = 11.32$$

$$T_2 = \frac{486.5}{11.32} = 43.1\,N$$

$$P = (T_1 - T_2)\,v = (486.5 - 43.1) \times 30$$

$$P = 13300\ W \text{ or } 13.3\,kW$$

No. of belts $= \dfrac{\text{Total power transmitted}}{\text{power " —— 1 belt}} = \dfrac{60}{13.3}$

$= 4.51$ or 5

length of the belt, $L_0 = \pi(R+r) + \dfrac{(R-r)^2}{C} + 2C = 4.79\,m$

Solution: Given : module $m = 6$ mm; addendum $a = 6$ mm

Gear ratio $G = 3 : 1$; $\Psi = 20°$; $N_{pinion} = 90$ r.p.m.

Here, as addendum = 1 × (module),

$$a_p = a_\omega = 1. \qquad \text{and} \qquad \omega_1 = \frac{2\pi \times 90}{60} = 9.43 \text{ rad/s}$$

Number of teeth required on gear to avoid interference,

$$T \geq \frac{2\, a_\omega}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\Psi} - 1} \qquad (1)$$

$$\geq \frac{2\,(1)}{\sqrt{1 + \frac{1}{3}\left(\frac{1}{3} + 2\right)\sin^2 20} - 1}; \text{ i.e., } \geq 44.94 = 45, \text{ say}$$

Therefore $\quad t \geq \frac{45}{3}$ i.e., $t \geq 15$

(*Note*: Since limiting condition of interference with standard module is reached first on the flank of pinion teeth, it is necessary to use relation for limiting number teeth on gear and then to find out limiting number of teeth on pinion.)

(b) length of path of approach

$$= \sqrt{R_a^2 - (R\cos\Psi)^2} - R\sin\Psi$$

But for $t = 15$ and $T = 45$, $\quad r = \frac{6 \times 15}{2} = 45$ mm and $R = \frac{6 \times 45}{2} = 135$ mm

and

$$r_a = 45 + 6 = 51 \text{ and } R = 135 + 6 = 141 \text{ mm}$$

Therefore length of path of approach

$$= \sqrt{(141)^2 - (135\cos 20)^2} - (135)\sin 20 = \underline{15.37} \text{ mm}$$

And, length of path of recess

$$= \sqrt{(51)^2 - (45\cos 20)^2} - (45)\sin 20 = \underline{13.12} \text{ mm.}$$

Therefore maximum velocity of sliding occurs at a point farthest (in the given case at point of engagement) from pitch point. Thus

$$(V_s)_{max} = 15.37\,(\omega_1 + \omega_2)$$

$$= 15.37\left(9.43 + \frac{9.43}{3}\right) = 193.25 \text{ mm/s}$$ **Ans.**

Length of path of contact = 15.37 + 13.12 = 28.49 mm **Ans.**

Length of arc of contact $= \frac{28.49}{\cos 20} = 30.32$ mm **Ans.**

No. of pairs of teeth in contact $= \frac{30.32}{P_c}$

Contact Ratio $= \frac{30.32}{m \cdot \pi} = \frac{30.32}{6\pi}$

$= 1.608$ Ans

Maximum velocity of sliding on one side = $(\omega_p + \omega_g)$ Path of approach

$\omega_p = \frac{2\pi \times 90}{60} = 9.425$ rad/s, $\omega_g = \frac{\omega_p}{3} = 3.142$ rad/s

$= \left(9.425 + \frac{9.425}{3}\right) \times 15.37 = 193.15$ mm/s

Velocity of sliding on other side = $(\omega_p + \omega_g)$ Path of recess $= \left(9.425 + \frac{9.425}{3}\right) \times 13.12$

$= 164.875$ mm/s

Velocity of sliding at pitch pt = $(\omega_p + \omega_g) \times 0 = 0$

.1 SIMPLE GEAR TRAIN → Classification of gear trains

A series of gears, capable of receiving and transmitting motion from one gear to another is called a simple gear train. In it, all the gear axes remain fixed relative to the frame and each gear is on a separate shaft (~~Fig. 11.1~~ Fig

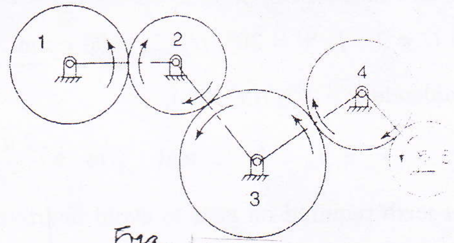


Fig a

In a simple gear train we can observe the following:

1. Two external gears of a pair always move in opposite directions.
2. All odd-numbered gears move in one direction and all even-numbered gears in the opposite direction. For example, gears 1, 3, 5, etc, move in the counter-clockwise direction.

T = No. of teeth on gear

N = Speed of gear-train in rpm

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \qquad \left[\text{Also } \frac{\omega_2}{\omega_1} = \frac{2\pi N_2}{2\pi N_1} = \frac{N_2}{N_1}\right]$$

$$\frac{N_3}{N_2} = \frac{T_2}{T_3}, \frac{N_4}{N_3} = \frac{T_3}{T_4} \text{ and } \frac{N_5}{N_4} = \frac{T_4}{T_5}$$

plying.

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} \times \frac{N_5}{N_4} = \frac{T_1}{T_2} \times \frac{T_2}{T_3} \times \frac{T_3}{T_4} \times \frac{T_4}{T_5}$$

$$\text{Train value } \frac{N_5}{N_1} = \frac{T_1}{T_5} = \frac{\text{number of teeth on driving gear}}{\text{number of teeth on driven gear}}$$

$$\text{Speed ratio} = \frac{1}{\text{train value}}$$

$$\frac{N_1}{N_5} = \frac{T_5}{T_1} \qquad (.1)$$

Thus, it is seen that the intermediate gears have no effect on the speed ratio and, therefore, they are known *idlers.*

.2 COMPOUND GEAR TRAIN

hen a series of gears are onnected in such a way that two r more gears rotate about an axis ith the same angular velocity, it is nown as compound gear train. In his type, some of the intermediate hafts, i.e., other than the input and he output shafts, carry more than one gear as shown in Fig. b.

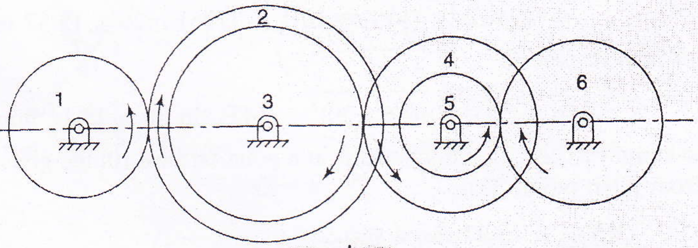


Fig. b

If the gear 1 is the driver then

$$\frac{N_2}{N_1} = \frac{T_1}{T_2}, \frac{N_4}{N_3} = \frac{T_3}{T_4} \text{ and } \frac{N_6}{N_5} = \frac{T_5}{T_6}$$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

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$$\frac{N_6}{N_1} = \frac{T_1}{T_2}\frac{T_3}{T_4}\frac{T_5}{T_6}$$

$$\text{Train value} = \frac{\text{product of number of teeth on driving gears}}{\text{product of number of teeth on driven gears}}$$

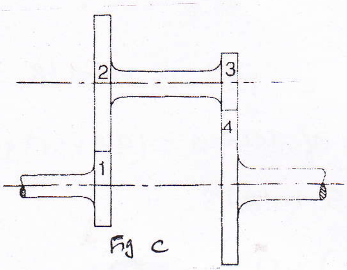


Fig c

.3 REVERTED GEAR TRAIN

If the axes of the first and the last wheels of a compound gear coincide it is called a reverted gear train. Such an arrangement is used in clocks and in simple lathes where *back gear* is used to give a slow speed to the chuck.

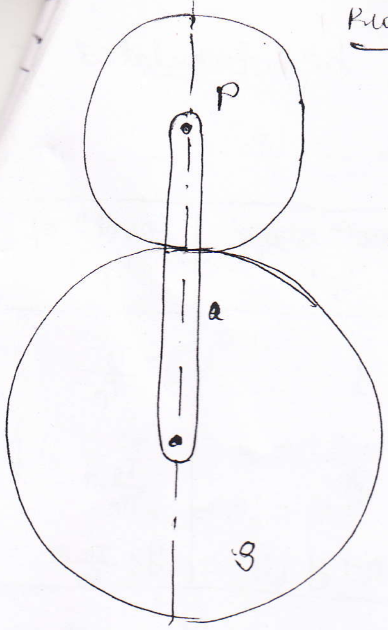
Referring ~~Fig. 11.~~4, Fig C

$$\frac{N_4}{N_1} = \frac{\text{product of number of teeth on driving gears}}{\text{product of number of teeth on driven gears}}$$

$$\frac{T_1}{T_2}\frac{T_3}{T_4} \qquad ~~(11.3)~~$$

Also, if r is the pitch circle radius of a gear,

$$r_1 + r_2 = r_3 + r_4$$



Procedure of Epicyclic Gear Train

A gear train having a relative motion of axis is called a planetary or an epicyclic gear train.

Two gear wheels S & P, the axes of which are connected by an arm a.

If the arm a is fixed, the wheels S & P constitute a simple train. If the wheel S is fixed so that the arm can rotate about the axes of S, the wheel P would also move around S.

∴ It is an epicyclic train.

They have complex motions

a - arm

P & S are gears.

Let the arm a be fixed & the wheel S be given x complete revolution in clockwise direction.

Then the wheel P will turn in anticlockwise direction through $\left(-\frac{T_S}{T_P}\right)x$ revolution.

Revolution made by $a = 0$

" ——— $S = x$

" ——— $P = -\left(\frac{T_S}{T_P}\right)x$

If the mechanism is locked together & turned through a no. of revolution, the relative motion bet^n a, S & P will not alter. Let the locked system be turned through y revolution in the clockwise direction

Revolution made by $a = y$

" ——— $S = x + y$

" ——— $P = y - \left(\frac{T_S}{\ }\right)x$

The total motion of each link may be tabulated following manner

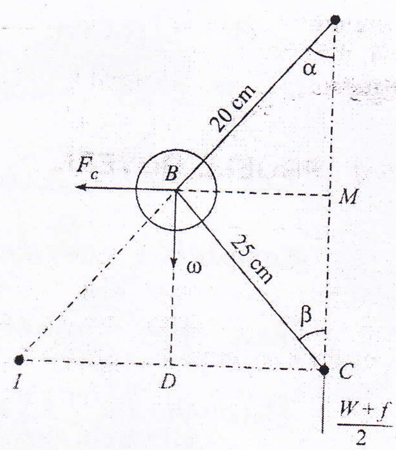
Line	Motion / Action	Revolution of Arm 'a'	Revolⁿ of 's'	Revolⁿ of 'p'
1	a fixed, s + 1 revolⁿ	0	1	$-\frac{T_S}{T_P}$
2	a — — , s + x — —	0	x	$-\frac{T_S}{T_P}x$
3	Add y	y	x + y	$y - \frac{T_S}{T_P}x$

let the arm make 1 revolⁿ clockwise when S is fixed

$y = 1 \quad , x + y = 0$

$x = -1$

Revolⁿ of $P = y - \left(\frac{T_S}{T_P}\right)x = 1 - \frac{T_S}{T_P}(-1) = 1 + \frac{T_S}{T_P}$ (5)



Solution: Given $W = 147.2$ N; $w = 19.6$ N

$f = 24.5$ N, $\alpha_1 = 40°$; $\alpha_2 = 30°$

(a) For minimum speed of rotation (*see* Fig. 15.10),

$\alpha_2 = 30°$; $r_2 = 20 \sin 30 = 10$ cm.

and, $h_2 = 20 \cos 30 = 17.32$ cm

also, $BM = r_2 = 25 \sin\beta_2$

Therefore $\sin\beta_2 = \frac{10}{25} = 0.4$ Hence, $\beta_2 = 23.578°$

Therefore $\tan\beta_2 = 0.436$ and $\tan\alpha_2 = 0.5773$

Thus $k_2 = 0.7559$

taking net load $= (w - f)/2$ on sleeve for obtaining lower most speed, we have

$$\omega_2^2 = \frac{w + (1 + k_2)(W - f)/2}{w} \times \frac{g}{h_2}$$

$$\omega_2^2 = \frac{19.6 + (1 + 0.7559)(147.2 - 24.5N)/2}{19.6} \times \frac{981}{17.32} = 367.9$$

therefore $\omega_2 = 19.18$ rad/s or, $N_2 = 183.2$ r.p.m.

(b) Similarly, for maximum speed of rotation N_1,

$\alpha_1 = 40°$ and $r_1 = 20 \sin 40 = 12.856$ cm

$h_1 = 20 \cos 40 = 15.32$ cm

Therefore $\tan\alpha_1 = 0.8391$

also $BM = r_1 = 25 \sin\beta_1$

Therefore $\sin\beta_1 = 12.856/25 = 0.5142$

thus $\beta_1 = 30.947°$ and $\tan\beta_1 = 0.5996$

Therefore $k_1 = \tan\beta_1/\tan\alpha_1 = 0.7146$

For maximum speed of rotation, taking sleeve load is $(W + f) = (147.2 + 24.5)$N

$$\omega_1^2 = \frac{w + (1 + k_1)(W + f)/2}{w} \times \frac{g}{h_1}$$

$$\omega_1^2 = \frac{19.6 + (1 + 0.7146)(147.2 + 24.5)/2}{\text{[illegible]}} \times \frac{981}{\text{[illegible]}} = 544.94$$

$\therefore\ \omega_1 = 23.34$ rad/s

$N_1 = 222.9$ rpm

Speed range $= 222.9 - 183.2$

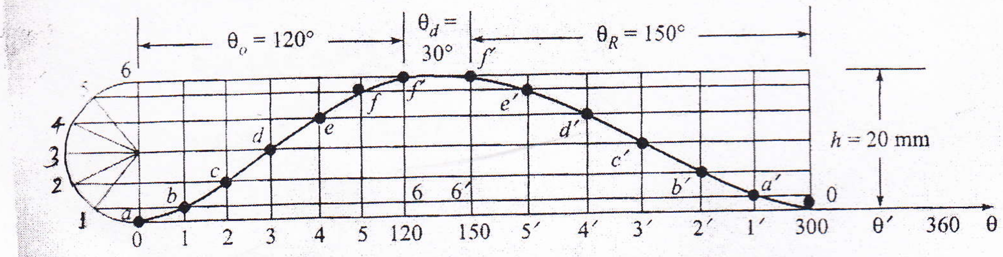
$= 39.7$ rpm   Ans

OR

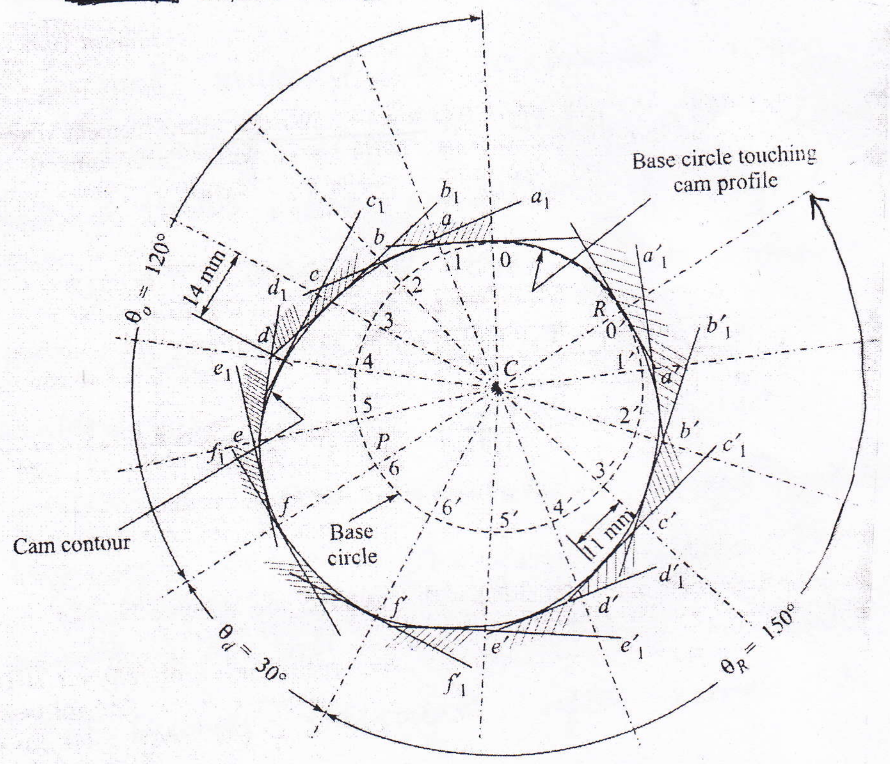
Q6 Given $h = 20$ mm.

$\phi_a = \theta_0 = 120^\circ$; $\phi_d = \theta_R = 150^\circ$

$\delta_1 = \theta_d = 30^\circ$; $\delta_2 = \theta'_d = 60^\circ$



[redacted] Displacement diagram for S.H.M. follower motion.



[redacted] Cam profile layout for flat faced follower.

$$V_{max} = \frac{\pi h \omega}{2\theta_0} = \frac{\pi(2.0)}{2(2\pi/3)} \times \left(\frac{2\pi \times 200}{60}\right) = 31.4159 \text{ cm/s}$$

and, $$A_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\theta_0}\right)^2$$

or, $$A_{max} = \frac{2.0}{2}\left(\frac{\pi}{2\pi/3}\right)^2 \omega^2 = 1.0\left(\frac{3}{2}\right)^2 \omega^2 = 987 \text{ cm/s}^2$$

(.1) SIMPLE GEAR TRAIN → Classification of Gears

A series of gears, capable of receiving and transmitting motion from one gear to another is called a simple gear train. In it, all the gear axes remain fixed relative to the frame and each gear is on a separate shaft (~~Fig. 11.1~~). Fig

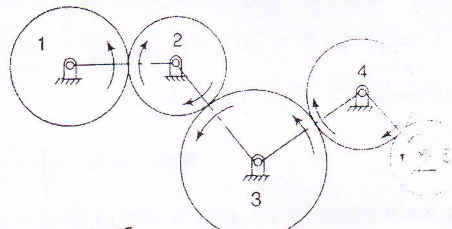


Fig a

In a simple gear train we can observe the following:

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2. All odd-numbered gears move in one direction and all even-numbered gears in the opposite direction. For example, gears 1, 3, 5, etc, move in the counter-clockwise direction.

T = No. of teeth on gear

N = Speed of gear-shaft in rpm

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \qquad \left[\text{Also } \frac{\omega_2}{\omega_1} = \frac{2\pi N_2}{2\pi N_1} = \frac{N_2}{N_1}\right]$$

$$\frac{N_3}{N_2} = \frac{T_2}{T_3},\ \frac{N_4}{N_3} = \frac{T_3}{T_4} \text{ and } \frac{N_5}{N_4} = \frac{T_4}{T_5}$$

...plying.

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} \times \frac{N_5}{N_4} = \frac{T_1}{T_2} \times \frac{T_2}{T_3} \times \frac{T_3}{T_4} \times \frac{T_4}{T_5}$$

$$\text{Train value } \frac{N_5}{N_1} = \frac{T_1}{T_5} = \frac{\text{number of teeth on driving gear}}{\text{number of teeth on driven gear}}$$

$$\text{Speed ratio} = \frac{1}{\text{train value}}$$

$$\frac{N_1}{N_5} = \frac{T_5}{T_1} \qquad (.1)$$

Thus, it is seen that the intermediate gears have no effect on the speed ratio and, therefore, they are known ... *idlers*.

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When a series of gears are connected in such a way that two or more gears rotate about an axis with the same angular velocity, it is known as compound gear train. In this type, some of the intermediate shafts, i.e., other than the input and the output shafts, carry more than one gear as shown in Fig. b.

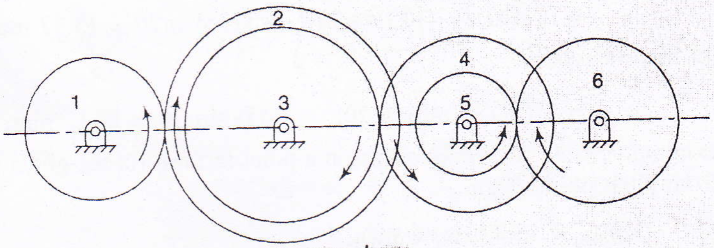

Fig. b

If the gear 1 is the driver then

$$\frac{N_2}{N_1} = \frac{T_1}{T_2},\ \frac{N_4}{N_3} = \frac{T_3}{T_4} \text{ and } \frac{N_6}{N_5} = \frac{T_5}{T_6}$$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

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$$\frac{N_6}{N_1} = \frac{T_1}{T_2} \frac{T_3}{T_4} \frac{T_5}{T_6}$$

$$\text{Train value} = \frac{\text{product of number of teeth on driving gears}}{\text{product of number of teeth on driven gears}}$$

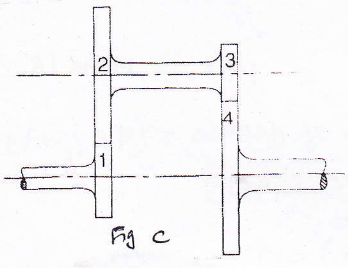


Fig c

(.3) REVERTED GEAR TRAIN

If the axes of the first and the last wheels of a compound gear coincide, it is called a reverted gear train. Such an arrangement is used in clocks and in simple lathes where *back gear* is used to give a slow speed to the chuck.

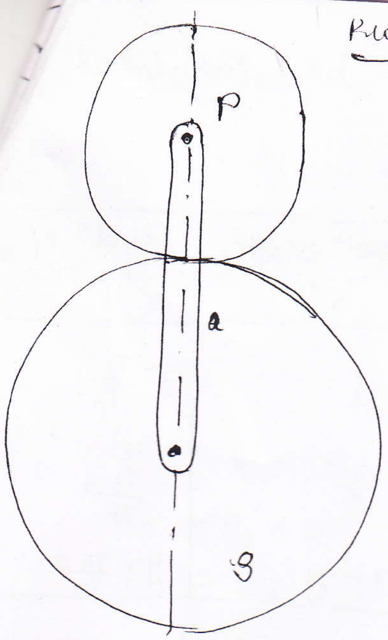
Referring ~~Fig. 11.~~4, Fig C

$$\frac{N_4}{N_1} = \frac{\text{product of number of teeth on driving gears}}{\text{product of number of teeth on driven gears}}$$

$$\frac{T_1}{T_2} \frac{T_3}{T_4}$$

Also, if r is the pitch circle radius of a gear,

$$r_1 + r_2 = r_3 + r_4$$



Procedure of Epicyclic Gear Train

A gear train having a relative motion of axis is called a planetary or an epicyclic gear train.

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If the arm a is fixed, the wheels S & P constitute a simple train. If the wheel S is fixed so that the arm can rotate about the axes of S, the wheel P would also move around S.

∴ It is an epicyclic train.

They have complex motions

a - arm

P & S are gears.

Let the arm a be fixed & the wheel S be given x complete revolution in clockwise direction.

Then the wheel P will turn in anticlockwise direction through $\left(-\frac{T_S}{T_P}\right)x$ revolution.

Revolution made by $a = 0$

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" ——————— $P = -\left(\frac{T_S}{T_P}\right)x$

If the mechanism is locked together & turned through a no. of revolution, the relative motion betn a, S & P will not alter. Let the locked system be turned through y revolution in the clockwise direction

Revolution made by $a = y$

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The total motion of each link may be tabulated following manner

Line	Motion / Action	Revolution of Arm 'a'	Revol^n of 's'	Revol^n of 'p'
1	a fixed, s + 1 revol^n	0	1	$-\frac{T_s}{T_p}$
2	a — — , s + x — —	0	x	$-\frac{T_s}{T_p}x$
3	Add y	y	$x+y$	$y-\frac{T_s}{T_p}x$

let the arm make 1 revol^n clockwise when s is fixed

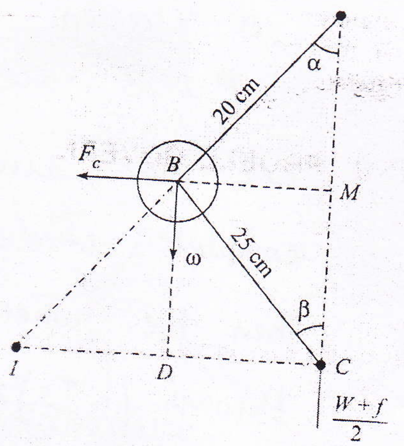
$y = 1 \quad , x + y = 0$

$x = -1$

Revol^n of $P = y - \left(\frac{T_s}{T_p}\right)x = 1 - \frac{T_s}{T_p}(-1) = 1 + \frac{T_s}{T_p}$ (5)

Solution: Given $W = 147.2$ N; $w = 19.6$ N

$f = 24.5$ N, $\alpha_1 = 40°$; $\alpha_2 = 30°$



(a) For minimum speed of rotation (*see* Fig. 15.10),

$\alpha_2 = 30°$; $r_2 = 20 \sin 30 = 10$ cm.

and, $h_2 = 20 \cos 30 = 17.32$ cm

also . $BM = r_2 = 25 \sin\beta_2$

Therefore $\sin\beta_2 = \frac{10}{25} = 0.4$ Hence, $\beta_2 = 23.578°$

Therefore $\tan\beta_2 = 0.436$ and $\tan\alpha_2 = 0.5773$

Thus $k_2 = 0.7559$

taking net load $= (w - f)/2$ on sleeve for obtaining lower most speed, we have

$$\omega_2^2 = \frac{w + (1+k_2)(W-f)/2}{w} \times \frac{g}{h_2}$$

$$\omega_2^2 = \frac{19.6 + (1+0.7559)(147.2 - 24.5\,N)/2}{19.6} \times \frac{981}{17.32} = 367.9$$

Therefore $\omega_2 = 19.18$ rad/s or, $N_2 = 183.2$ r.p.m.

(b) Similarly, for maximum speed of rotation N_1,

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$N_1 = 222.9$ rpm

Speed range $= 222.9 - 183.2$

$= 39.7$ rpm Ans

OR

Q6 Given $h = 20$ mm.

$\phi_a = \theta_0 = 120^\circ$; $\phi_d = \theta_R = 150^\circ$

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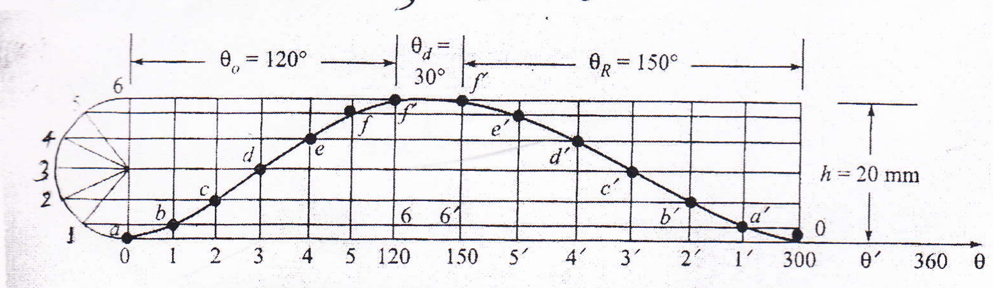
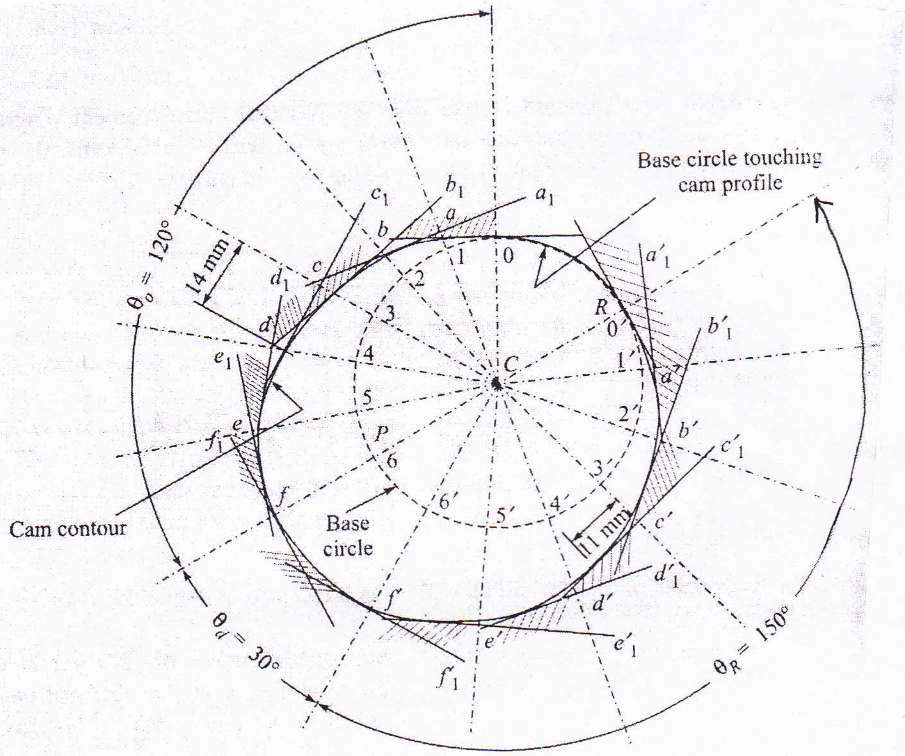


[redacted] Displacement diagram for S.H.M. follower motion.



[redacted] Cam profile layout for flat faced follower.

$$V_{max} = \frac{\pi h \omega}{2\theta_0} = \frac{\pi(2.0)}{2(2\pi/3)} \times \left(\frac{2\pi \times 200}{60}\right) = 31.4159 \text{ cm/s}$$

and, $$A_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\theta_0}\right)^2$$

or, $$A_{max} = \frac{2.0}{2}\left(\frac{\pi}{2\pi/3}\right)^2 \omega^2 = 1.0\left(\frac{3}{2}\right)^2 \omega^2 = 987 \text{ cm/s}^2$$